

### Exercise 4A

$$\begin{aligned} 1 \text{ Work done} &= Fs \\ &= 0.6 \times 4.2 \\ &= 2.52 \end{aligned}$$

The work done is 2.52 J

$$\begin{aligned} 2 \text{ Work done} &= Fs \\ 102 &= F \times 12 \\ F &= \frac{102}{12} = 8.5 \end{aligned}$$

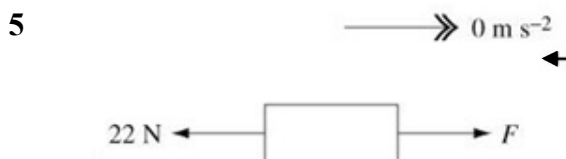
The magnitude of the force is 8.5 N

$$\begin{aligned} 3 \text{ Work done against gravity} &= mgh \\ &= 0.35 \times 9.8 \times 7 \\ &= 24.01 \end{aligned}$$

The work done against gravity is 24.0 J (3 s.f.)

$$\begin{aligned} 4 \text{ Work done against gravity} &= mgh \\ &= 15 \times 9.8 \times 4 \\ &= 588 \end{aligned}$$

The work done against gravity is 588 J



No acceleration, so the force pushing the box has the same magnitude as the resistances.

$$F = 22 \text{ N}$$

$$\begin{aligned} \text{Work done} &= Fs \\ &= 22 \times 15 \\ &= 330 \end{aligned}$$

The work done by the force pushing the box is 330 J

$$\begin{aligned} 6 \text{ Work done by gravity} &= mgh \\ &= 0.5 \times 9.8 \times 15 \\ &= 73.5 \end{aligned}$$

The work done by gravity is 73.5 J

7 Work done =  $mgh$   
 $30 \times 1000 = 80 \times 9.8 h$

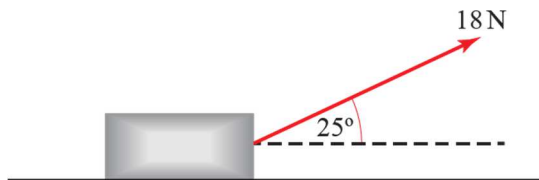
1 kJ = 1000 J

$$h = \frac{30000}{80 \times 9.8}$$

$$h = 38.26\dots$$

The building is 38.3 m high (3 s.f.)

8 a

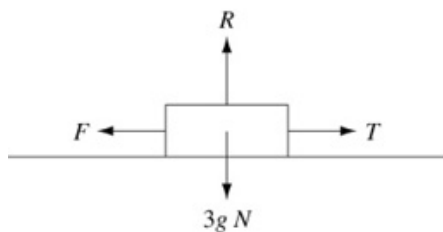


Work done = horizontal component of force  $\times$  distance moved  
 $= 18 \cos 25^\circ \times 14$   
 $= 228.38\dots$

The work done is 228 J (3 s.f.)

- b One assumption made is that there is no frictional force between the sled and the ice.  
 This is likely to be a valid assumption, due to the low coefficient of friction between sled and ice.

9



Work done =  $Ts$   
 $30 = T \times 4$   
 $T = 7.5$

Resolving parallel to the plane:

$$T - F = 0$$

$$7.5 - F = 0$$

$$F = 7.5$$

Resolving perpendicular to the plane to find  $R$ .

$$R = mg$$

$$R = 3 \times 9.8$$

Friction is limiting:

$$F = \mu R$$

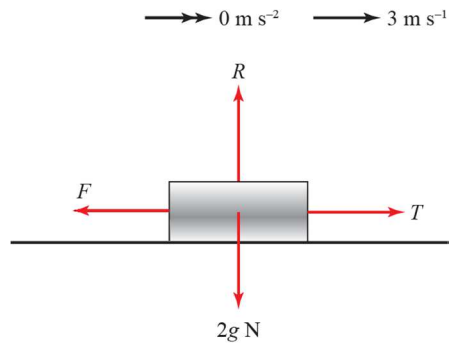
$$7.5 = \mu \times 3 \times 9.8$$

$$\mu = \frac{7.5}{3 \times 9.8} = 0.2551\dots$$

The coefficient of friction is 0.255 (3 s.f.)

The parcel moves at a constant speed so the acceleration is  $0 \text{ m s}^{-2}$

10



$$\mu = 0.55$$

Resolving perpendicular to the plane:

$$R = 2g$$

Friction is limiting:

$$F = \mu R$$

$$F = 0.55 \times 2g$$

Resolving parallel to the plane:

$$T - F = 0$$

$$T = 0.55 \times 2g$$

$$\text{Work done} = Ts$$

$$= 0.55 \times 2g \times (3 \times 2)$$

$$= 0.55 \times 2 \times 9.8 \times 6$$

$$= 64.68$$

The work done is 64.7 J (3 s.f.)

Distance moved = speed  $\times$  time

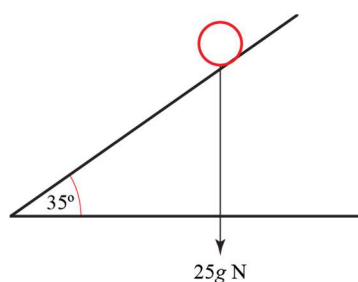
11 Work done against gravity =  $mgh$ 

$$= 52 \times 9.8 \times 46$$

$$= 23\,441.6$$

The work done against gravity is 23 400 J (3 s.f.)

12

Work done by gravity =  $mgh$ 

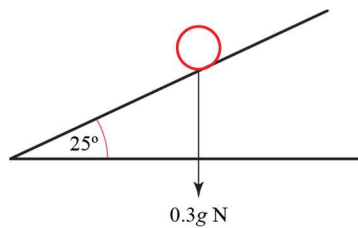
$$= 25 \times 9.8 \times (2 \sin 35^\circ)$$

$$= 281.0\dots$$

The work done by gravity is 281 J (3 s.f.)

Vertical distance  
moved =  $2 \sin 35^\circ$

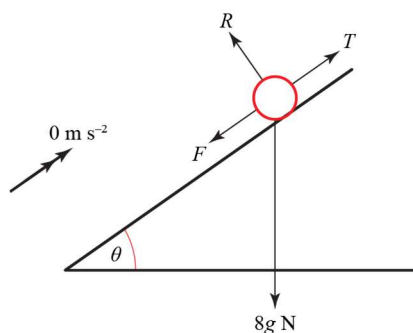
13



$$\begin{aligned} \text{Work done against gravity} &= mgh \\ &= 0.3 \times 9.8 \times (2 \sin 25^\circ) \\ &= 2.484\dots \end{aligned}$$

The work done against gravity is 2.48 J (3 s.f.)

14



$$\mu = 0.3$$

**a** Resolving perpendicular to the plane:

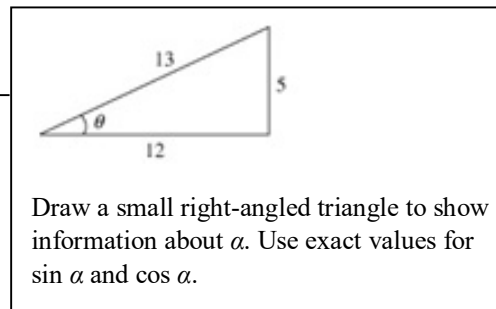
$$\begin{aligned} R &= 8g \cos \alpha \\ &= 8g \times \frac{12}{13} \end{aligned}$$

Friction is limiting:

$$F = \mu R$$

$$\begin{aligned} F &= 0.3 \times 8 \times 9.8 \times \frac{12}{13} \\ &= 21.71\dots \end{aligned}$$

The frictional force has magnitude 21.7 N (3 s.f.)



**b** Work done against friction =  $Fs$

$$\begin{aligned} &= 21.71\dots \times 15 \\ &= 325.6\dots \end{aligned}$$

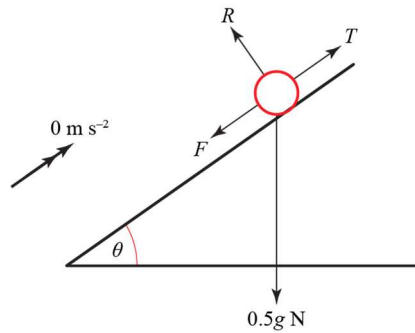
The work done against friction is 326 J (3 s.f.)

**c** Work done against gravity =  $mgh$

$$\begin{aligned} &= 8 \times 9.8 \times (15 \sin \alpha) \\ &= 8 \times 9.8 \times \left( 15 \times \frac{5}{13} \right) \\ &= 452.3\dots \end{aligned}$$

The work done against gravity is 452 J (3 s.f.)

15



Resolving perpendicular to the plane:

$$R = 0.5g \cos \theta$$

$$= 0.5g \times \frac{24}{25}$$

Resolving parallel to the plane:

$$T = F + 0.5g \sin \theta$$

Friction is limiting:

$$F = \mu R$$

$$F = \mu \times 0.5g \times \frac{24}{25}$$

$$T = \mu \times 0.5g \times \frac{24}{25} + 0.5g \times \frac{7}{25}$$

Work done by force = force  $\times$  distance moved

$$12 = T \times 3$$

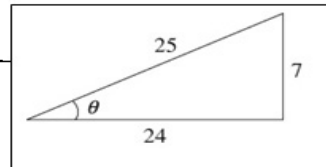
$$T = 4$$

$$\therefore 4 = \mu \times 0.5g \times \frac{24}{25} + 0.5g \times \frac{7}{25}$$

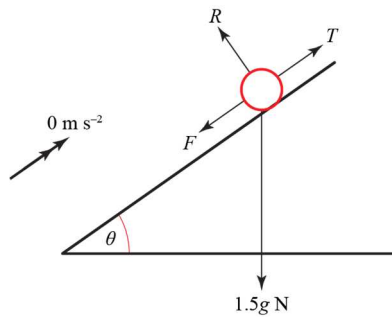
$$\mu = \frac{4 - 0.5 \times 9.8 \times \frac{7}{25}}{0.5 \times 9.8 \times \frac{24}{25}}$$

$$\mu = 0.5586\dots$$

The coefficient of friction is 0.559 (3 s.f.)



16



$$\mu = 0.4$$

Resolving perpendicular to the plane:

$$R = 1.5g \cos 40^\circ$$

Friction is limiting:

$$F = \mu R$$

$$F = 0.4 \times 1.5g \cos 40^\circ$$

Resolving parallel to the plane:

$$T = F + 1.5g \sin 40^\circ$$

$$T = 0.4 \times 1.5g \cos 40^\circ + 1.5g \sin 40^\circ$$

Work done by  $T = T \times s$ 

$$= (0.4 \times 1.5g \cos 40^\circ + 1.5g \sin 40^\circ) \times 8$$

$$= 111.6\dots$$

The work done by  $T$  is 112 J (3 s.f.)

$$17 \sin \alpha = \frac{3}{5} \Rightarrow \cos \alpha = \frac{4}{5}$$

Work done = force  $\times$  distance moved in direction of force**a** Work done by gravity  $E_g = Wh$ 

$$\text{Weight, } W = mg = 2g, h = 3 \sin \alpha = \frac{9}{5} \text{ m}$$

$$E_g = 2g \times \frac{9}{5}$$

$$E_g = 2 \times 9.8 \times 1.8 = 35.28$$

The work done by gravity is 35.3 J (3 s.f.)

**b** Work done by friction  $E_F = Fs$ ,  $s = 3$  mNormal reaction force,  $R$ , can be found by resolving perpendicular to the slope:

$$R = 2g \cos \alpha$$

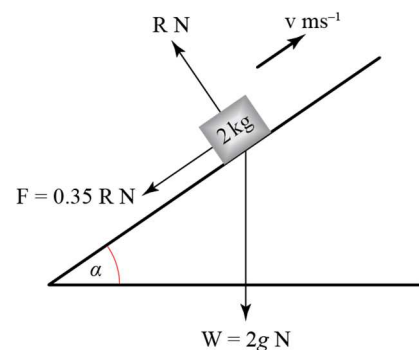
$$R = \frac{8}{5}g$$

$$\text{So frictional force, } F = \frac{7}{20} \times \frac{8}{5}g = \frac{14}{25}g$$

$$E_F = \frac{14}{25}g \times 3$$

$$E_F = 0.56 \times 9.8 \times 3 = 16.464$$

The work done by gravity is 16.5 J (3 s.f.)



**17 c** Work done against these forces = kinetic energy lost

Since kinetic energy =  $\frac{1}{2}mv^2$ , so here:

$$35.28 + 16.464 = \left(\frac{1}{2} \times 2u^2\right) - \left(\frac{1}{2} \times 2 \times 0^2\right)$$

$$51.744 = u^2$$

$$u = 7.1933\dots$$

The particle is projected at a speed of  $7.19 \text{ m s}^{-1}$  (3 s.f.)

## Exercise 4B

1 a Kinetic energy =  $\frac{1}{2}mv^2 = \frac{1}{2} \times 0.3 \times 15^2 = 33.8 \text{ J (3 s.f.)}$

b Kinetic energy =  $\frac{1}{2}mv^2 = \frac{1}{2} \times 3 \times 2^2 = 6 \text{ J}$

c Kinetic energy =  $\frac{1}{2}mv^2 = \frac{1}{2} \times 0.1 \times 100^2 = 500 \text{ J}$

d Kinetic energy =  $\frac{1}{2}mv^2 = \frac{1}{2} \times 25 \times 4^2 = 200 \text{ J}$

e Kinetic energy =  $\frac{1}{2}mv^2 = \frac{1}{2} \times 800 \times 20^2 = 160\,000 \text{ J}$

In order, from the most kinetic energy to the least, will be **e, c, d, a, b**

2 a Gain of P.E. =  $mgh = 1.5 \times 9.8 \times 3 = 44.1 \text{ J}$

b Gain of P.E. =  $mgh = 55 \times 9.8 \times 15 = 8085 \text{ J}$

c Loss of P.E. =  $mgh = 75 \times 9.8 \times 30 = 22\,050 \text{ J}$

d Loss of P.E. =  $mgh = 580 \times 9.8 \times 6 = 34\,104 \text{ J}$

3 Decrease in K.E. =  $\frac{1}{2}mu^2 - \frac{1}{2}mv^2$   
 $= \frac{1}{2} \times 1.2 \times 12^2 - \frac{1}{2} \times 1.2 \times 4^2$   
 $= 76.8$

The decrease in the particle's K.E. is 76.8 J

4 Increase in K.E. =  $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$   
 $= \frac{1}{2} \times 900 \times 20^2 - \frac{1}{2} \times 900 \times 5^2$   
 $= 168\,750$

The increase in the van's K.E. is 168 750 J

5 Increase in K.E. =  $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$   
 $6 = \frac{1}{2} \times 0.2 \times v^2 - \frac{1}{2} \times 0.2 \times 2^2$   
 $6 = 0.1v^2 - 0.4$   
 $v^2 = \frac{6.4}{0.1} = 64$   
 $v = 8 \quad (v > 0)$

← Speed is positive.

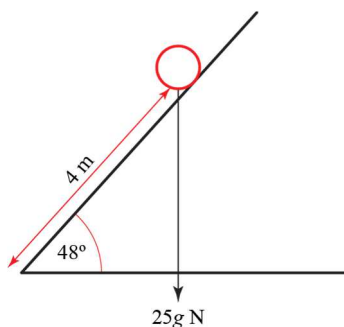
The value of  $v$  is 8.



$$\begin{aligned}
 6 \quad \text{Decrease in K.E.} &= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 \\
 100 &= \frac{1}{2} \times 45 \times 5^2 - \frac{1}{2} \times 45v^2 \\
 100 &= 562.5 - 22.5v^2 \\
 v^2 &= \frac{462.5}{22.5} \\
 v &= \pm 4.533\dots \\
 v &= 4.533\dots \quad (v > 0)
 \end{aligned}$$

The skater's final speed is  $4.53 \text{ m s}^{-1}$  (3 s.f.)

7 a



$$\begin{aligned}
 \text{P.E. lost} &= mgh \\
 &= 25 \times 9.8 \times (4 \sin 48^\circ) \\
 &= 728.2\dots
 \end{aligned}$$

Vertical distance moved is  $4 \sin 48^\circ$ .

The P.E. lost by the child is 728 J (3 s.f.)

b You have assumed there to be no air resistance. This would be valid for low speeds, but not for high speeds.

8 a  $s = 2 \text{ m}$ ,  $a = 9.8 \text{ m s}^{-2}$ ,  $u = 0$ ,  $v = ?$

$$v^2 = u^2 + 2as$$

$$v^2 = 0 + 2 \times 9.8 \times 2$$

$$v^2 = 39.2$$

$$\begin{aligned}
 \text{K.E.} &= \frac{1}{2}mv^2 = \frac{1}{2} \times 0.6 \times 39.2 \\
 &= 11.76
 \end{aligned}$$

Use  $v^2 = u^2 + 2as$  to find the speed of the ball as it hits the water.

The K.E. of the ball as it hits the surface of the water is 11.8 J (3 s.f.)

$$\begin{aligned}
 \text{b K.E. lost} &= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 \\
 &= 11.76 - \frac{1}{2} \times 0.6 \times 4.8^2 \\
 &= 4.848
 \end{aligned}$$

The K.E. lost by the ball is 4.85 J (3 s.f.)

9  $u = 35 \text{ m s}^{-1}$ ,  $a = -1.2 \text{ m s}^{-2}$ ,  $t = 5 \text{ s}$ ,  $v = ?$

$$v = u + at$$

$$v = 35 - 1.2 \times 5$$

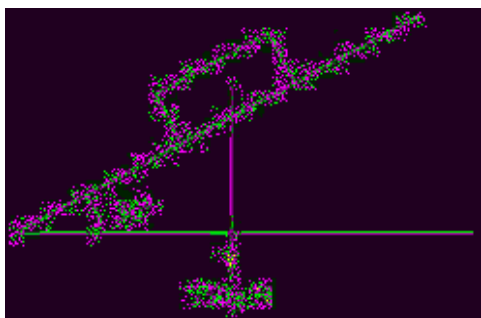
$$v = 29$$

$$\begin{aligned} \text{Loss of K.E.} &= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 2000 \times 35^2 - \frac{1}{2} \times 2000 \times 29^2 \\ &= 384\,000 \end{aligned}$$

The loss of K.E. of the lorry is 384 000 J

Use  $v = u + at$  to find the final speed of the lorry.

10



a Loss of K.E.  $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

$$\begin{aligned} &= \frac{1}{2} \times 750 \times 20^2 - \frac{1}{2} \times 750 \times 15^2 \\ &= 65\,625 \end{aligned}$$

The loss of K.E. of the car is 65 625 J

b Gain of P.E.  $= mgh$

$$\begin{aligned} &= 750 \times 9.8 \times (500 \sin 30^\circ) \\ &= 1\,837\,500 \end{aligned}$$

The gain of P.E. of the car is 1 837 500 J

11 Increase of P.E.  $= mgh$

$$15.7 \times 1000 = 80 \times 9.8h$$

$$h = \frac{15.7 \times 1000}{80 \times 9.8}$$

$$h = 20.02$$

The cliff is 20.0 m high (3 s.f.)

1 kJ = 1000 J

### Challenge

- a** The ball is dropped from the top of a cliff, and falls freely under gravity.

Use the equation  $v = u + at$

Using  $u = 0$  and  $a = g$ , you have  $v = gt$

$$\text{K.E.} = \frac{1}{2}mv^2 = \frac{1}{2} \times 1 \times (gt)^2 = \frac{g^2t^2}{2} = 48.0t^2$$

$s = ut + \frac{1}{2}at^2$ . So using  $s = h$ ,  $u = 0$  and  $a = g$ , you have  $h = \frac{1}{2}gt^2$

$$\text{P.E.} = -mgh = -1 \times g \times \left(\frac{1}{2}gt^2\right) = -\frac{g^2t^2}{2} = -48.0t^2$$

- b** Kinetic energy + potential energy =  $\frac{g^2t^2}{2} + \left(-\frac{g^2t^2}{2}\right) = 0$

So kinetic energy + potential energy is constant.

## Exercise 4C

1 a P.E. lost =  $mgh = 0.4 \times 9.8 \times 7$   
 $= 27.44$

The P.E. lost is 27.4 J (3 s.f.)

b K.E. gained =  $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$   
 $= \frac{1}{2} \times 0.4 \times v^2 - 0$

P.E. lost = K.E. gained

$$27.44 = \frac{1}{2} \times 0.4 \times v^2$$

$$v^2 = \frac{27.44}{0.2}$$

$$v = 11.71\dots$$

The final speed of the particle is 11.7 m s<sup>-1</sup> (3 s.f.)

2 a K.E. gained =  $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$   
 $= \frac{1}{2} \times 0.5 \times 12^2 - 0$   
 $= 36$

The K.E. gained by the stone is 36 J

b P.E. lost = K.E. gained  
 $= 36$  J

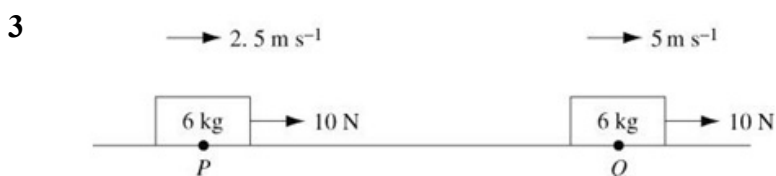
The P.E. lost by the stone is 36 J

c P.E. lost =  $mgh$   
 $36 = 0.5 \times 9.8 \times h$

$$h = \frac{36}{0.5 \times 9.8}$$

$$h = 7.346\dots$$

The height of the tower is 7.35 m (3 s.f.)



a Increase in K.E. =  $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$   
 $= \frac{1}{2} \times 6 \times 5^2 - \frac{1}{2} \times 6 \times 2.5^2$   
 $= 56.25$

The increase in K.E. of the box is 56.3 J (3 s.f.)

b The work done by the force is 56.3 J

3 c  $F = ma$   
 $10 = 6a$

$$a = \frac{5}{3}$$

Substituting into:

$$v^2 = u^2 + 2as$$

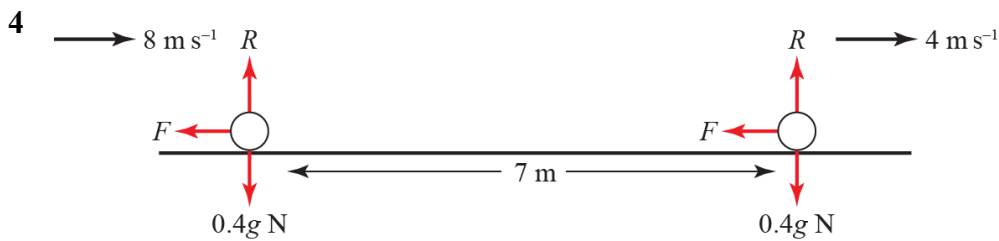
With  $u = 2.5 \text{ m s}^{-1}$ ,  $v = 5 \text{ m s}^{-1}$  and  $a = \frac{5}{3} \text{ m s}^{-2}$  gives:

$$5^2 = 2.5^2 + 2\left(\frac{5}{3}\right)s$$

$$25 = 6.25 + \frac{10}{3}s$$

$$s = 5.625 \text{ m}$$

$$s = 5.63 \text{ m (3 s.f.)}$$



a K.E. lost  $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$   
 $= \frac{1}{2} \times 0.4 \times 8^2 - \frac{1}{2} \times 0.4 \times 4^2$   
 $= 9.6$

The K.E. lost by the particle is 9.6 J

b The work done against friction is 9.6 J Work done = change in energy

c Resolving perpendicular to the surface:  $R = 0.4g$

Friction is limiting:  $F = \mu R$

$$F = 0.4g \times \mu$$

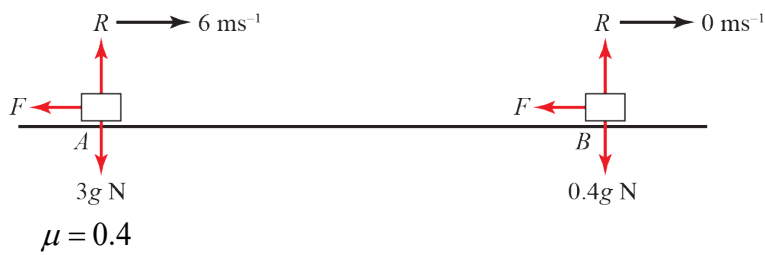
Work done  $= Fs$

$$9.6 = 0.4g \times \mu \times 7$$

$$\mu = \frac{9.6}{0.4 \times 9.8 \times 7} = 0.3498\dots$$

The coefficient of friction is 0.350 (3 s.f.)

5



**a** K.E. lost  $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$   
 $= \frac{1}{2} \times 3 \times 6^2 - 0$   
 $= 54$

The kinetic energy lost by the box is 54 J

**b** The work done against friction is 54 J

**c** Resolving perpendicular to the floor:  $R = 3g$

Friction is limiting:  $F = \mu R$

$$F = 0.4 \times 3g$$

Work done  $= Fs$

$$54 = 0.4 \times 3g \times s$$

$$s = \frac{54}{0.4 \times 3g} = 4.591\dots$$

The distance  $AB$  is 4.59 m (3 s.f.)

**6** P.E. lost  $= mgh$

$$= 0.8 \times 9.8 \times 5$$

$$= 39.2$$

K.E. gained  $=$  P.E. lost

$$= 39.2$$

$$\text{K.E. gained} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$39.2 = \frac{1}{2} \times 0.8v^2 - 0$$

$$v^2 = \frac{39.2 \times 2}{0.8}$$

$$v = 9.899\dots$$

The particle hits the ground at a speed of 9.90 m s<sup>-1</sup> (3 s.f.)

$$\begin{aligned}
 7 \text{ K.E. gained} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\
 &= \frac{1}{2} \times 0.3 \times 20^2 - 0 \\
 &= 60
 \end{aligned}$$

$$\begin{aligned}
 \text{P.E. lost} &= \text{K.E. gained} \\
 &= 60
 \end{aligned}$$

$$\text{P.E. lost} = mgh$$

$$60 = 0.3 \times 9.8 \times h$$

$$h = \frac{60}{0.3 \times 9.8}$$

$$h = 20.40\dots$$

The cliff is 20.4 m high (3 s.f.)

$$\begin{aligned}
 8 \text{ P.E. gained} &= mgh \\
 &= 0.3 \times 9.8 \times 5
 \end{aligned}$$

$$\text{K.E. lost} = \text{initial K.E.} - \text{final K.E.}$$

$$= \frac{1}{2} \times mu^2 - 2.1$$

$$= \frac{1}{2} \times 0.3u^2 - 2.1$$

$$\text{K.E. lost} = \text{P.E. gained}$$

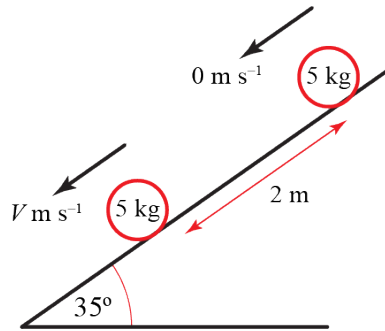
$$\frac{1}{2} \times 0.3u^2 - 2.1 = 0.3 \times 9.8 \times 5$$

$$u^2 = \frac{0.3 \times 9.8 \times 5 + 2.1}{\frac{1}{2} \times 0.3}$$

$$u = 10.58\dots$$

The value of  $u$  is 10.6 (3 s.f.)

9



**a** P.E. lost =  $mgh$   
 $= 5 \times 9.8 \times (2 \sin 35^\circ)$   
 $= 56.21\dots$   
The P.E. lost is  $56.2\text{ J}$  (3 s.f.)

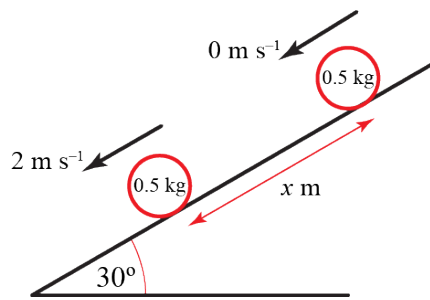
**b** The K.E. gained is  $56.2\text{ J}$

**c** K.E. gained =  $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$   
 $56.21 = \frac{1}{2} \times 5 \times v^2 - 0$   
 $v^2 = \frac{56.21 \times 2}{5}$   
 $v = 4.741\dots$

The final speed of the package is  $4.74\text{ m s}^{-1}$  (3 s.f.)



10



$$\begin{aligned}\text{K.E. gained} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= \frac{1}{2} \times 0.5 \times 2^2 - 0 \\ &= 1\end{aligned}$$

$$\text{P.E. lost} = mgh = 0.5 \times 9.8 \times (x \sin 30^\circ)$$

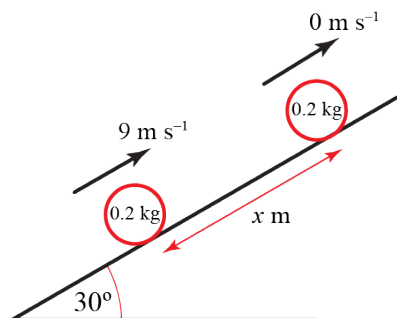
$$\text{P.E. lost} = \text{K.E. gained}$$

$$0.5 \times 9.8 \times (x \sin 30^\circ) = 1$$

$$\begin{aligned}x &= \frac{1}{0.5 \times 9.8 \times \sin 30^\circ} \\ &= 0.4081\dots\end{aligned}$$

The value of  $x$  is 0.408 (3 s.f.)

11



$$\begin{aligned} \text{K.E. lost} &= \frac{1}{2} mu^2 - \frac{1}{2} mv^2 \\ &= \frac{1}{2} \times 0.2 \times 9^2 - 0 \end{aligned}$$

$$\begin{aligned} \text{P.E. gained} &= mgh \\ &= 0.2 \times 9.8 \times (x \sin 30^\circ) \end{aligned}$$

$$\text{P.E. gained} = \text{K.E. lost}$$

$$\begin{aligned} 0.2 \times 9.8 \times (x \sin 30^\circ) &= \frac{1}{2} \times 0.2 \times 9^2 \\ x &= \frac{\frac{1}{2} \times 0.2 \times 9^2}{0.2 \times 9.8 \sin 30^\circ} \\ &= 8.265\dots \end{aligned}$$

The value of  $x$  is 8.27 (3 s.f.)

$$\begin{aligned} \text{12 K.E. lost} &= \frac{1}{2} mu^2 - \frac{1}{2} mv^2 \\ &= \frac{1}{2} \times 0.6 u^2 - 0 \end{aligned}$$

$$\begin{aligned} \text{P.E. gained} &= mgh \\ &= 0.6 \times 9.8 \times (5 \sin 40^\circ) \end{aligned}$$

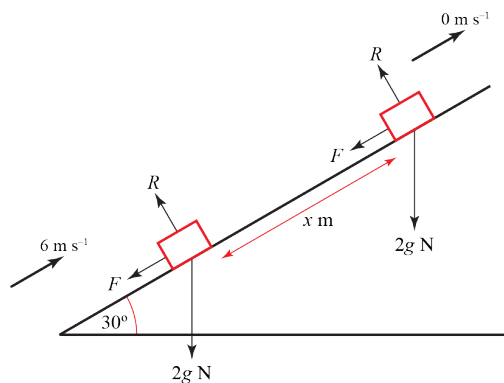
$$\text{K.E. lost} = \text{P.E. gained}$$

$$\begin{aligned} \frac{1}{2} \times 0.6 u^2 &= 0.6 \times 9.8 \times 5 \sin 40^\circ \\ u^2 &= \frac{0.6 \times 9.8 \times 5 \sin 40^\circ}{\frac{1}{2} \times 0.6} \end{aligned}$$

$$u = 7.936\dots$$

The speed of projection is  $7.94 \text{ m s}^{-1}$  (3 s.f.)

13



$$\begin{aligned} \text{K.E. lost} &= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 2 \times 6^2 - 0 \\ &= 36 \end{aligned}$$

$$\begin{aligned} \text{P.E. gained} &= mgh \\ &= 2 \times 9.8 \times (x \sin 30^\circ) \\ &= 9.8x \end{aligned}$$

Resolving perpendicular to the plane:  $R = 2g \cos 30^\circ$

Friction is limiting:  $F = \mu R$

$$F = \frac{1}{3} \times 2g \cos 30^\circ = \frac{2}{3}g \cos 30^\circ$$

Work done against friction =  $Fx = \frac{2}{3}gx \cos 30^\circ$

K.E. lost = P.E. gained + work done against friction

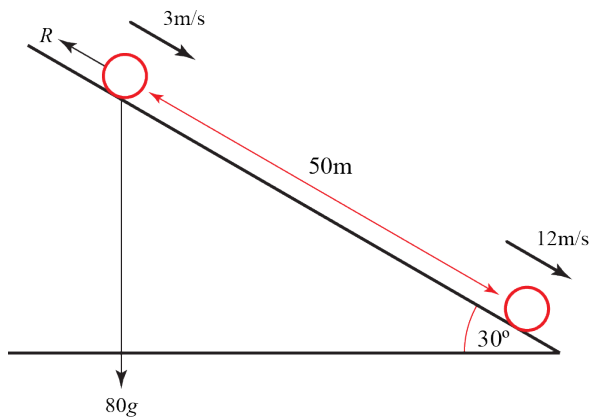
$$\Rightarrow 36 = 9.8x + \frac{2}{3}gx \cos 30^\circ$$

$$36 = 9.8x \left( 1 + \frac{2}{3} \cos 30^\circ \right)$$

$$x = \frac{36}{9.8 \left( 1 + \frac{2}{3} \cos 30^\circ \right)} = 2.328\dots$$

The particle moves 2.33 m up the plane (3 s.f.)

14



a Work done by resistive forces on the skier = change in total energy of the skier

$$\text{Loss in P.E.} = mgh$$

$$\text{Increase in K.E.} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Total loss of energy = P.E. lost - K.E. gained

$$= mgh + \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$

$$\text{Force} \times \text{distance} = mgh + \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$

$$50R = (80 \times 9.8 \times 50 \sin 30^\circ) + \left(\frac{1}{2} \times 80 \times 3^2\right) - \left(\frac{1}{2} \times 80 \times 12^2\right)$$

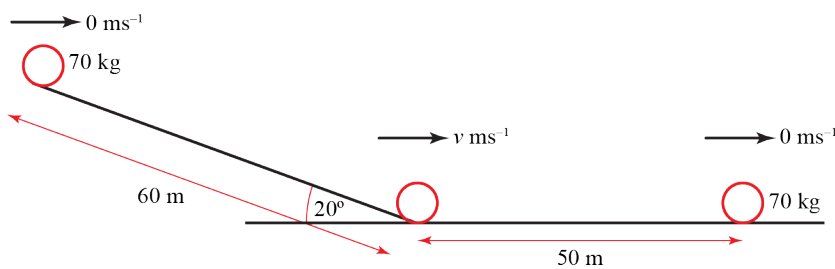
$$50R = 14\,200$$

$$R = 284$$

The value of  $R$  is 284.

b The resistive force may not be constant, and could depend on speed, for example.

15



$$\begin{aligned} \text{Change in K.E.} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= 0 - 0 \end{aligned}$$

Loss of P.E. =  $mgh$

$$= 70 \times 9.8 \times (60 \sin 20^\circ)$$

Work done against resistance =  $F_S$

$$= R \times (60 + 50)$$

$$= 110R$$

Work done against resistance = loss of P.E.

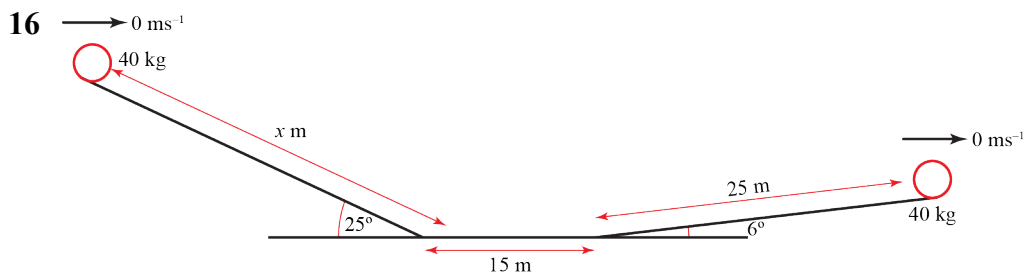
$$110R = 70 \times 9.8 \times (60 \sin 20^\circ)$$

$$R = \frac{70 \times 9.8 \times 60 \sin 20^\circ}{110}$$

$$R = 127.9\dots$$

The value of  $R$  is 128 (3 s.f.)

Consider energy changes from start to end – do not divide the motion into two parts.



$$\begin{aligned} \text{Loss of P.E.} &= mgh \\ &= 40 \times 9.8 \times (x \sin 25^\circ - 25 \sin 6^\circ) \end{aligned}$$

$$\begin{aligned} \text{Change in K.E.} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= 0 - 0 \end{aligned}$$

$$\begin{aligned} \text{Work done against resistance} &= Fs \\ &= 18 \times (x + 15 + 25) \\ &= 18 \times (x + 40) \end{aligned}$$

$$\begin{aligned} \text{Work done against resistance} &= \text{loss of P.E.} \\ 18x + 18 \times 40 &= 40 \times 9.8 \times x \sin 25^\circ - 40 \times 9.8 \times 25 \sin 6^\circ \end{aligned}$$

$$(40 \times 9.8 \sin 25^\circ - 18)x = 18 \times 40 + 40 \times 9.8 \times 25 \sin 6^\circ$$

$$x = \frac{18 \times 40 + 40 \times 9.8 \times 25 \sin 6^\circ}{40 \times 9.8 \sin 25^\circ - 18}$$

$$x = 11.81 \dots$$

The girl travels 11.8 m down the slope.

Consider energy changes from start to end – do not divide the motion into three parts.

### Challenge

Let the mass of a hydrogen molecule =  $m$

So the mass of an oxygen molecule =  $8m$

Consider the average kinetic energy of the oxygen molecules:

$$\frac{1}{2}mv^2 = \frac{1}{2} \times 8m \times 400^2 = \frac{3}{2}kT$$

Consider the average kinetic energy of the hydrogen molecules:

$$\text{Average K.E.} = \frac{3}{2}kT = \frac{1}{2} \times 8m \times 400^2 = \frac{1}{2}mv^2$$

$$\text{So } \frac{1}{2} \times 8m \times 400^2 = \frac{1}{2}mv^2$$

$$8 \times 400^2 = v^2$$

$$v = \sqrt{1\,280\,000}$$

$$= 1131.3 \dots$$

The average speed of the hydrogen molecules is  $1130 \text{ m s}^{-1}$

## Exercise 4D

1

$$\begin{aligned} \text{Power} &= Fv \\ &= 1500 \times 12 \\ &= 18\,000 \end{aligned}$$

The power is 18 kW

2 Power =  $Fv$ 

$$\begin{aligned} &= 1000 \times 15 \\ &= 15\,000 \end{aligned}$$

The power is 15 000 W (or 15 kW)

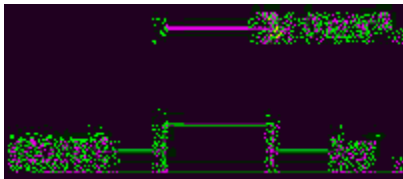
3 Power =  $Fv$ 

$$5000 = F \times 18$$

$$\begin{aligned} F &= \frac{5000}{18} \\ &= 277.7\dots \end{aligned}$$

The driving force has magnitude 278 N (3 s.f.)

4



At maximum speed the acceleration is zero.

Resolving horizontally:  $T = 600$

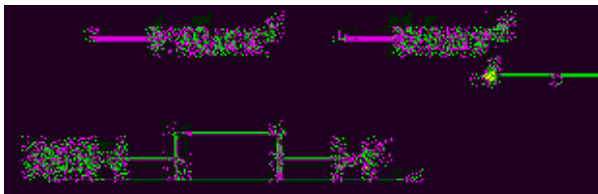
Power =  $Fv$

$$15\,000 = 600v$$

$$\begin{aligned} v &= \frac{15\,000}{600} \\ &= 25 \end{aligned}$$

The maximum speed is  $25 \text{ m s}^{-1}$

5 a



At maximum speed the acceleration is zero.

Resolving horizontally:  $T = 500$

Power =  $Fv$

$$= 500 \times 40$$

$$= 20\,000$$

The power is 20 000 W (or 20 kW)

**b** The resistance to motion of the car would typically be expected to increase with speed but it would be reasonable to assume a constant resistive force if the car maintained the same speed and the gradient and surface of the road stayed the same.

6  $\longrightarrow \longrightarrow 16 \text{ m s}^{-1}$   $\longrightarrow \longrightarrow 0 \text{ m s}^{-2}$



$$\text{Power} = Fv$$

$$8.8 \times 10^3 = T \times 16$$

$$T = \frac{8800}{16}$$

$$T = 550$$

Resolving horizontally:  $R = T$

$$R = 550$$

The magnitude of the resistance is 550 N

7 a  $\longrightarrow \longrightarrow 7 \text{ m s}^{-1}$   $\longrightarrow \longrightarrow a \text{ m s}^{-2}$



$$\text{Power} = Fv$$

$$9000 = T \times 7$$

$$T = \frac{9000}{7}$$

Now using  $F = ma$  :

$$T - 350 = ma$$

$$\frac{9000}{7} - 350 = 850a$$

$$a = \frac{\frac{9000}{7} - 350}{850}$$

$$a = 1.100\dots$$

The acceleration is  $1.10 \text{ m s}^{-2}$  (3 s.f.)

First find the tractive force produced by the engine and then use  $F = ma$  to find the acceleration.

7 b  $\longrightarrow 15 \text{ m s}^{-1} \quad \longrightarrow \longrightarrow a \text{ m s}^{-2}$



$$\text{Power} = Fv$$

$$9000 = T \times 15$$

$$T = \frac{9000}{15} = 600$$

Now using  $F = ma$  :

$$T - 350 = ma$$

$$600 - 350 = 850a$$

$$a = \frac{250}{850}$$

$$a = 0.2941\dots$$

The acceleration is  $0.294 \text{ m s}^{-2}$  (3 s.f.)

c  $\longrightarrow v \text{ m s}^{-1} \quad \longrightarrow \longrightarrow v \text{ m s}^{-2}$



Resolving horizontally:  $T = 350$

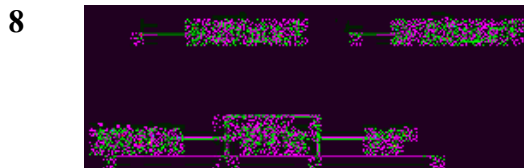
$$\text{Power} = Fv$$

$$9000 = 350v$$

$$v = \frac{9000}{350}$$

$$v = 25.71\dots$$

The maximum speed is  $25.7 \text{ m s}^{-1}$  (3 s.f.)



Using  $F = ma$ :

$$T - 300 = 900 \times 0.3$$

$$T = 900 \times 0.3 + 300$$

$$= 570$$

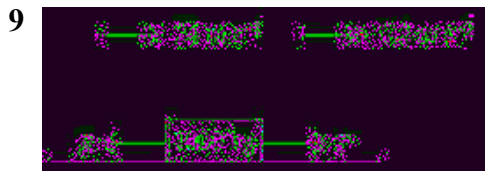
$$\text{Power} = Fv$$

$$= 570 \times 20$$

$$= 11\,400$$

The power development by the engine is  $11\,400 \text{ W}$  (or  $11.4 \text{ kW}$ )





$$\text{Power} = Fv$$

$$12\,000 = T \times 24$$

$$T = \frac{12\,000}{24} = 500$$

Using  $F = ma$ :

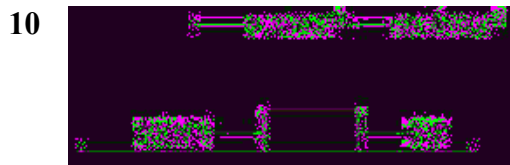
$$T - R = 1000 \times 0.2$$

$$500 - R = 200$$

$$R = 500 - 200$$

$$R = 300$$

The value of  $R$  is 300.



Resolving horizontally:

$$T = 28$$

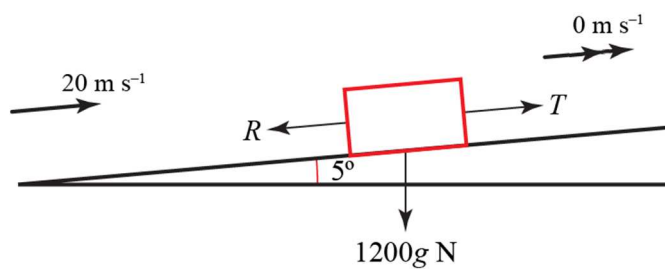
$$\text{Power} = Fv$$

$$280 = 28v$$

$$v = 10$$

The cyclist's maximum speed is  $10 \text{ m s}^{-1}$

11 a



$$\text{Power} = Fv$$

$$24\,000 = T \times 20$$

$$T = \frac{24\,000}{20} = 1200$$

Resolving parallel to the slope:

$$T = R + 1200g \sin 5^\circ$$

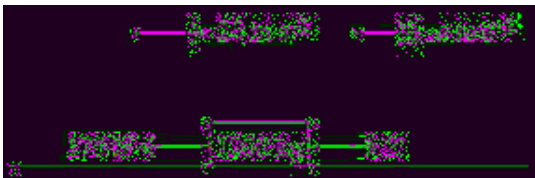
$$1200 = R + 1200g \sin 5^\circ$$

$$R = 1200 - 1200g \sin 5^\circ$$

$$R = 175.04\dots$$

The value of  $R$  is 175 (3 s.f.)

11 b



From part a,  $T = 1200 \text{ N}$

Using  $F = ma$ :

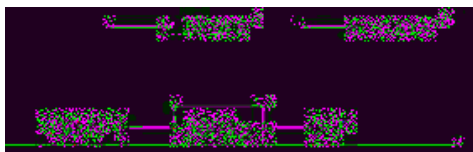
$$1200 - 175 = 1200a$$

$$a = \frac{1200 - 175}{1200}$$

$$a = 0.8541\dots$$

The initial acceleration of the van is  $0.854 \text{ m s}^{-2}$  (3 s.f.)

12 a



Power =  $Fv$

$$26000 = T \times 18$$

$$T = \frac{26000}{18}$$

Using  $F = ma$ :

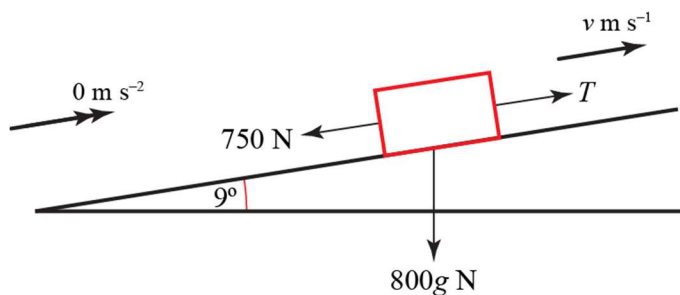
$$T - 750 = 800a$$

$$800a = \frac{26000}{18} - 750$$

$$a = 0.8680\dots$$

The acceleration is  $0.868 \text{ m s}^{-2}$  (3 s.f.)

b



Resolving parallel to the slope:

$$T = 750 + 800g \sin 9^\circ$$

Power =  $Fv$

$$26000 = T \times v$$

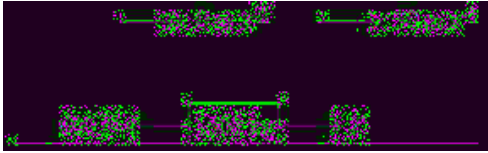
$$26000 = (750 + 800 \times 9.8 \sin 9^\circ)v$$

$$v = \frac{26000}{(750 + 800 \times 9.8 \sin 9^\circ)}$$

$$v = 13.15\dots$$

The maximum speed is  $13.2 \text{ m s}^{-1}$  (3 s.f.)

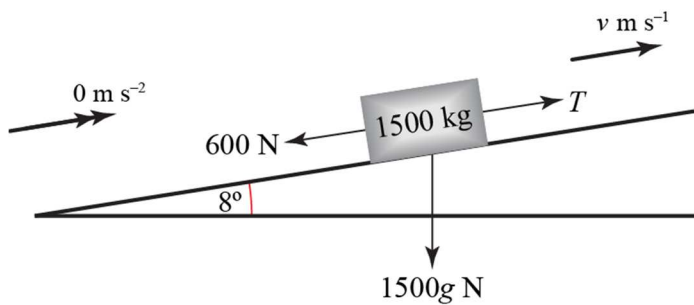
13 a

Resolving horizontally:  $T = 600$ 

$$\begin{aligned} \text{Power} &= Fv \\ &= 600 \times 30 \\ &= 18000 \end{aligned}$$

The power is 18 000 W (or 18 kW)

b



Resolving parallel to the slope:

$$T = 600 + 1500g \sin 8^\circ$$

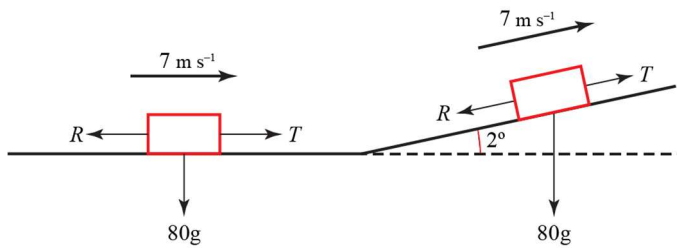
Power =  $Fv$ 

$$18000 = (600 + 1500g \sin 8^\circ)v$$

$$\begin{aligned} v &= \frac{18000}{(600 + 1500g \sin 8^\circ)} \\ &= 6.803\dots \end{aligned}$$

The maximum speed is  $6.80 \text{ m s}^{-1}$  (3 s.f.)

14



Consider the cyclist on level ground:

$$\text{Power} = Tv$$

$$\text{Power} = 7T$$

Since the velocity is constant, resolving horizontally:

$$T = R$$

$$\text{So power} = 7R$$

Consider the cyclist cycling uphill:

$$\text{Power} = Tv$$

$$\text{Power} = 7T$$

Since the velocity is constant, resolving parallel to the plane:

$$T = R + 80g \sin 2^\circ$$

$$\text{So power} = 7T = 7(R + 80g \sin 2^\circ) = 7R + 560g \sin 2^\circ$$

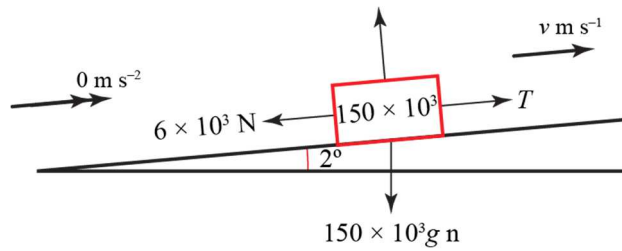
Therefore, the increase in power required is:

$$(7R + 560g \sin 2^\circ) - 7R = 560g \sin 2^\circ$$

$$= 191.5\dots$$

The increase in power required is 192 W (3 s.f.)

15 a



Resolving parallel to the slope:

$$T = (6 \times 10^3) + (150 \times 10^3 g \sin 2^\circ)$$

Power =  $Fv$ 

$$350 \times 10^3 = (6 \times 10^3 + 150 \times 10^3 g \sin 2^\circ) \times v$$

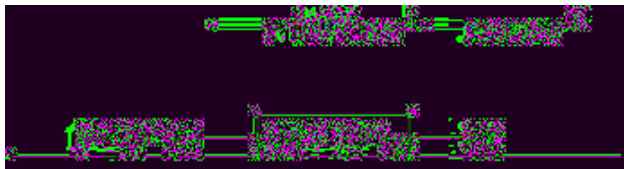
$$v = \frac{350}{(6 + 150 \times 9.8 \sin 2^\circ)}$$

$$= 6.107\dots$$

The maximum speed is  $6.11 \text{ m s}^{-1}$ 

1 tonne =  $10^3$  kg. When tonnes, kilonewtons and kilowatts are used the  $10^3$  will cancel, leaving easier numbers.

b

Power =  $Fv$ 

$$350 \times 10^3 = T \times 6.107$$

$$T = \frac{350 \times 10^3}{6.107}$$

Using  $F = ma$ :

$$T - 6 \times 10^3 = 150 \times 10^3 \times a$$

$$\frac{350 \times 10^3}{6.107} - 6 \times 10^3 = 150 \times 10^3 \times a$$

$$150a = \frac{350}{6.107} - 6$$

$$a = 0.3420\dots$$

The initial acceleration is  $0.342 \text{ m s}^{-2}$  (3 s.f.)

16  $\longrightarrow \longrightarrow 0 \text{ m s}^{-2} \longrightarrow v \text{ m s}^{-1}$



$$\text{Power} = 10 \text{ kW} = 10\,000 \text{ W}$$

$$\text{Power} = Tv$$

$$10\,000 = Tv$$

$$T = \frac{10\,000}{v}$$

When the velocity is maximum, the acceleration =  $0 \text{ m s}^{-2}$

Therefore the resultant force is 0 N. Resolving horizontally:

$$T = 150 + 3v$$

$$\text{So } \frac{10\,000}{v} = 150 + 3v$$

$$10\,000 = 150v + 3v^2$$

Rearranging:

$$3v^2 + 150v - 10\,000 = 0$$

Using the quadratic formula:

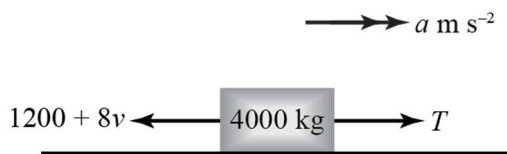
$$v = \frac{-150 \pm \sqrt{150^2 - 4 \times 3 \times (-10\,000)}}{6}$$

$$v = \frac{-150 \pm \sqrt{142\,500}}{6}$$

$$\text{Since } v > 0, v = \frac{-150 + \sqrt{142\,500}}{6} = 37.91\dots$$

The maximum value of  $v$  is  $37.9 \text{ m s}^{-1}$  (3 s.f.)

17 a



$$\text{Power} = 28 \text{ kW} = 28\,000 \text{ W}$$

$$\text{Power} = Tv$$

$$28\,000 = Tv$$

$$T = \frac{28\,000}{v}$$

Resolving horizontally and using  $F = ma$  :

$$\frac{28\,000}{v} - (1200 + 8v) = 4000a$$

When  $v = 10 \text{ m s}^{-1}$ :

$$2800 - (1200 + 80) = 4000a$$

$$1520 = 4000a$$

$$\text{So } a = \frac{1520}{4000} = 0.38 \text{ m s}^{-2}$$

b Using  $P = Fv$ , when the car is travelling at speed  $w$ :

$$28\,000 = Tw$$

$$T = \frac{28\,000}{w}$$

Resistive force =  $1200 + 8w$

As in part a, resolving horizontally and using  $F = ma$ :

$$\frac{28\,000}{w} - (1200 + 8w) = 4000a$$

When  $v = w \text{ m s}^{-1}$ ,  $a = -0.2 \text{ m s}^{-2}$ :

$$\frac{28\,000}{w} - (1200 + 8w) = 4000 \times (-0.2)$$

$$28\,000 - 1200w - 8w^2 = -800w$$

$$8w^2 + 400w - 28\,000 = 0$$

$$w^2 + 50w - 3500 = 0$$

Using the quadratic formula:

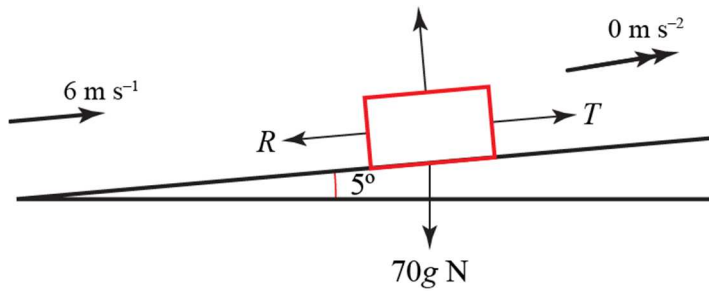
$$w = \frac{-50 \pm \sqrt{50^2 - 4 \times 1 \times (-3500)}}{2} = \frac{-50 \pm \sqrt{16\,500}}{2}$$

$$\text{Since } w > 0, w = \frac{-50 + \sqrt{16\,500}}{2} = 39.22\dots$$

The value of  $w$  is 39.2 (3 s.f.)

## Chapter Review

1



$$\text{Power} = Fv$$

$$480 = T \times 6$$

$$T = \frac{480}{6} = 80$$

Resolving parallel to the slope:

$$T = R + 70g \sin 5^\circ$$

$$80 = R + 70 \times 9.8 \sin 5^\circ$$

$$R = 80 - 70 \times 9.8 \sin 5^\circ$$

$$R = 20.21\dots$$

The magnitude of the resistance is 20.2 N (3 s.f.)

$$\begin{aligned} 2 \text{ a } \text{P.E. gained by water and bucket} &= mgh \\ &= 12 \times 9.8 \times 25 \\ &= 2940 \end{aligned}$$

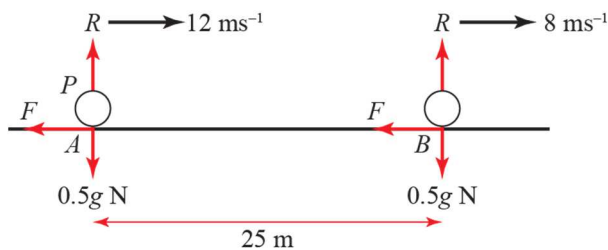
$$\text{Initial K.E.} = \text{final K.E.} = 0$$

$$\begin{aligned} \text{Work done by the boy} &= \text{P.E. gained by bucket} \\ &= 2940 \text{ J} \end{aligned}$$

$$\begin{aligned} \text{b } \text{Average rate of working} &= \frac{\text{work done}}{\text{time taken}} = \frac{2940}{30} \\ &= 98 \end{aligned}$$

The average rate of working of the boy is  $98 \text{ J s}^{-1}$  (or 98 W)

3



$$\begin{aligned} \text{a } \text{K.E. lost by particle} &= \frac{1}{2} \times 0.5 \times 12^2 - \frac{1}{2} \times 0.5 \times 8^2 \\ &= 20 \end{aligned}$$

$$\text{Work done by friction} = \text{K.E. lost by particle}$$

$$\therefore \text{Work done by friction} = 20 \text{ J}$$



- 3 b Resolving vertically:  $R = 0.5g$

Friction is limiting:

$$F = \mu R = \mu \times 0.5g$$

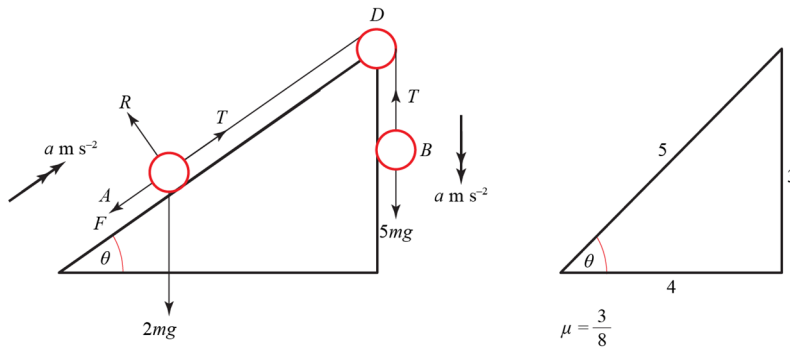
Work done by friction =  $F \times s$

$$20 = \mu \times 0.5g \times 25$$

$$\mu = \frac{20}{0.5g \times 25} = 0.1632\dots$$

The coefficient of friction is 0.163 (3 s.f.)

4



- a Resolving perpendicular to the plane for A:

$$R = 2mg \cos \theta$$

Friction is limiting:

$$F = \mu R$$

$$F = \frac{3}{8} \times 2mg \cos \theta$$

$$= \frac{3}{8} \times 2mg \times \frac{4}{5}$$

$$= \frac{3}{5} mg$$

$$F = ma \text{ for } A: \quad T - (F + 2mg \sin \theta) = 2ma$$

$$T - \left( \frac{3}{5} mg + 2mg \times \frac{3}{5} \right) = 2ma$$

$$T - \frac{9mg}{5} = 2ma \quad (1)$$

$$F = ma \text{ for } B: \quad 5mg - T = 5ma \quad (2)$$

$$(1) + (2): \quad 5mg - \frac{9mg}{5} = 7ma$$

$$\frac{16mg}{5} = 7ma$$

$$a = \frac{16g}{35} = \frac{16 \times 9.8}{35}$$

$$a = 4.48$$

The initial acceleration of A is  $4.48 \text{ m s}^{-2}$

4 b For the first 1 m  $A$  travels ←

The motion must be considered in two parts, before and after the string breaks. The friction force acting on  $A$  is the same throughout the motion.

$$u = 0$$

$$a = 4.48 \text{ m s}^{-2}$$

$$s = 1 \text{ m}$$

$$v = ?$$

$$v^2 = u^2 + 2as$$

$$v^2 = 2 \times 4.48 \times 1$$

$$v^2 = 8.96$$

After string breaks:

$$\begin{aligned} \text{Loss of K.E. (of } A) &= \frac{1}{2} mu^2 - \frac{1}{2} mv^2 \\ &= \frac{1}{2} \times 2m \times 8.96 - 0 \\ &= 8.96m \end{aligned}$$

$$\begin{aligned} \text{Gain of P.E. (of } A) &= mgh \\ &= 2mg \times (x \sin \theta) \\ &= 2mg \times x \times \frac{3}{5} \\ &= \frac{6mgx}{5} \end{aligned}$$

where  $x$  is the distance moved up the plane.

$$\text{Work done by friction} = \frac{3mg}{5} \times x$$

Work–energy principle:

$$\begin{aligned} \frac{3mgx}{5} + \frac{6mgx}{5} &= 8.96m \\ \frac{9gx}{5} &= 8.96 \end{aligned}$$

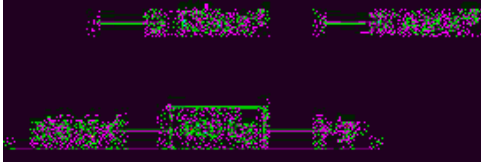
$$x = \frac{8.96 \times 5}{9 \times 9.8}$$

$$x = 0.5079\dots$$

$$\begin{aligned} \text{Total distance moved} &= 1 + 0.5079\dots \\ &= 1.51 \end{aligned}$$

The total distance moved by  $A$  before it first comes to rest is 1.51 m (3 s.f.)

5 a



$$\text{Power} = Fv$$

$$16000 = T \times 15$$

$$T = \frac{16000}{15}$$

Using  $F = ma$  :

$$T - 500 = 800a$$

$$\frac{16000}{15} - 500 = 800a$$

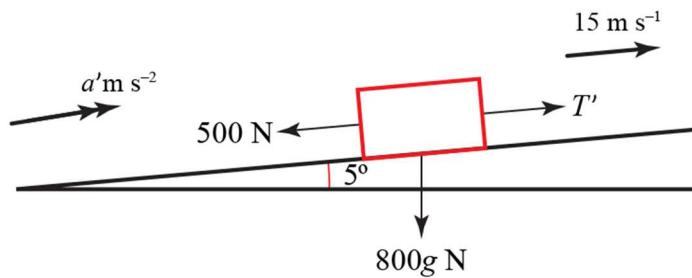
$$a = \frac{\frac{16000}{15} - 500}{800}$$

$$a = 0.7083\dots$$

Ensure units are consistent.

The acceleration is  $0.708 \text{ m s}^{-2}$ 

b



$$\text{Power} = Fv$$

$$24000 = T' \times 15$$

$$T' = \frac{24000}{15}$$

Resolving parallel to the slope and using  $F = ma$  :

$$T' - 500 - 800g \sin 5^\circ = 800a'$$

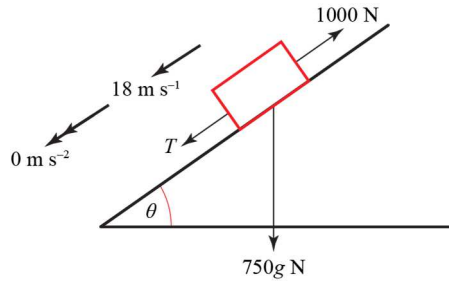
$$\frac{24000}{15} - 500 - 800 \times 9.8 \sin 5^\circ = 800a'$$

$$800a' = 416.698\dots$$

$$a' = 0.5208\dots$$

The new acceleration is  $0.521 \text{ m s}^{-2}$  (3 s.f.)

6 a



$$\tan \theta = \frac{1}{20} \quad \text{so} \quad \theta = 2.8624^\circ$$

Resolving parallel to the slope:

$$T + 750g \sin \theta = 1000$$

$$T = 1000 - 750 \times 9.8 \sin 2.8624^\circ$$

$$T = 632.95$$

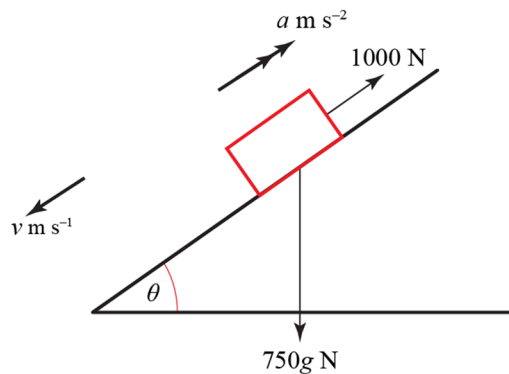
Power =  $Fv$ 

$$= 632.95 \times 18$$

$$= 11393.2 \dots$$

The rate of working of the car's engine is 11.4 kW (3 s.f.)

b



The tractive force is zero.

Resolving parallel to the slope and using  $F = ma$ :

$$1000 - 750 \times 9.8 \times \sin \theta = 750a$$

$$a = \frac{1000 - 750 \times 9.8 \sin 2.8624^\circ}{750}$$

$$a = 0.8439$$

Consider motion down the slope:

$$a = -0.8439 \text{ m s}^{-2}, \quad u = 18 \text{ m s}^{-1}, \quad v = 0 \text{ m s}^{-1}, \quad t = ?$$

$$v = u + at$$

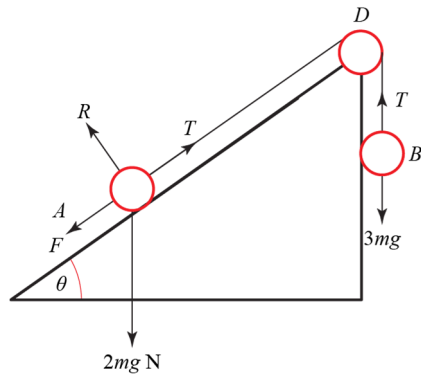
$$0 = 18 - 0.8439 \times t$$

$$t = \frac{18}{0.8439}$$

$$t = 21.32 \dots$$

The value of  $t$  is 21.3 (3 s.f.)

7



$$\begin{aligned}
 \text{a P.E. gained by } A &= mgh \\
 &= 2mg \times (s \times \sin \theta) \\
 &= 2mg \times s \times \frac{3}{5} \\
 &= \frac{6mgs}{5}
 \end{aligned}$$

$$\begin{aligned}
 \text{P.E. lost by } B &= mgh \\
 &= 3mgs
 \end{aligned}$$

$$\therefore \text{ P.E. lost by system} = 3mgs - \frac{6mgs}{5} = \frac{9mgs}{5}$$

7 b Consider  $A$ :

Find the frictional force and use the work–energy principle.

Resolving perpendicular to the slope:

$$\begin{aligned} R &= 2mg \cos \theta \\ &= 2mg \times \frac{4}{5} \\ &= \frac{8mg}{5} \end{aligned}$$

Friction is limiting:

$$\begin{aligned} F &= \mu R \\ &= \frac{1}{4} \times \frac{8mg}{5} \\ &= \frac{2mg}{5} \end{aligned}$$

Work done against friction =  $Fs$

$$= \frac{2mgs}{5}$$

K.E. gained by  $A$  and  $B = \frac{1}{2}(2m)v^2 + \frac{1}{2}(3m)v^2$

$$= \frac{5mv^2}{2}$$

Work–energy principle:

K.E. gained + work done against friction = P.E. lost

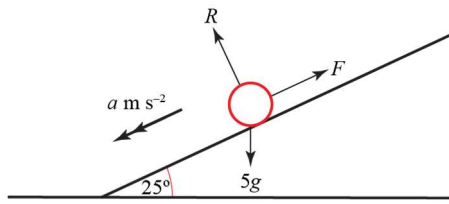
$$\frac{5mv^2}{2} + \frac{2mgs}{5} = \frac{9mgs}{5}$$

$$\frac{5mv^2}{2} = \frac{7mgs}{5}$$

$$v^2 = \frac{2 \times 7mgs}{5 \times 5m}$$

$$v^2 = \frac{14gs}{25}$$

8 a



Resolving parallel to the slope and using  $F = ma$ :

$$5g \sin 25^\circ - F = 5a$$

Friction is limiting:

$$F = \mu R$$

$$F = 0.3 \times 5g \cos 25^\circ$$

$$\text{So } 5g \sin 25^\circ - 5 \times 0.3 \times g \cos 25^\circ = 5a$$

$$a = g(\sin 25^\circ - 0.3 \cos 25^\circ)$$

Consider the motion down the slope.

$$u = 0 \text{ and } t = 2$$

$$v = u + at$$

$$= 0 + 2g(\sin 25^\circ - 0.3 \cos 25^\circ)$$

$$= 2g(\sin 25^\circ - 0.3 \cos 25^\circ)$$

$$= 2.9542 \dots$$

After it has been moving for 2 s the parcel has speed  $2.95 \text{ m s}^{-1}$  (3 s.f.)

**b** In 2 s the parcel slides a distance  $s$  m down the sloping platform.

$$\text{Loss of P.E.} = mgh$$

$$= mg \times s \sin 25^\circ$$

$$= 5g \times s \sin 25^\circ$$

$$u = 0, v = 2.954 \text{ m s}^{-1}, t = 2 \text{ s}$$

$$\text{Using } s = \frac{u+v}{2} \times t$$

$$s = \frac{0 + 2.954}{2} \times 2 = 2.954$$

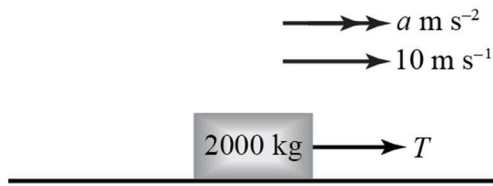
$$\text{So, loss of P.E.} = 5g \times 2.954 \times \sin 25^\circ$$

$$= 5 \times 9.8 \times 2.954 \times \sin 25^\circ$$

$$= 61.17 \dots$$

During the 2 s, the parcel loses 61.2 J of potential energy (3 s.f.)

9



$$\text{Power} = 4000 \text{ W}$$

$$\text{Power} = Tv = 10T$$

$$\text{So } T = \frac{4000}{10} = 400 \text{ N}$$

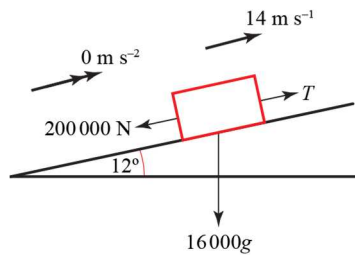
Using  $F = ma$ :

$$T = 2000 \times a$$

$$400 = 2000a$$

$$\text{So } a = \frac{400}{2000} = 0.2 \text{ m s}^{-2}$$

10



Resolving parallel to the slope:

$$T = 200000 + 16000g \sin 12^\circ$$

$$T = 232600.5 \dots$$

Work done in 10 s = force  $\times$  distance moved

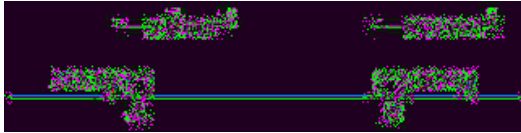
$$= 232600 \dots \times (14 \times 10)$$

$$= 32564000 \text{ (3 s.f.)}$$

The work done in 10s is 32 600 000 J (or 32 600 kJ) (3 s.f.)



11



$$\begin{aligned} \text{a K.E. gained} &= \frac{1}{2}mv^2 - \frac{1}{2}mu^2 \\ &= \frac{1}{2} \times 0.3 \times 12^2 - \frac{1}{2} \times 0.3 \times 6^2 \\ &= 16.2 \end{aligned}$$

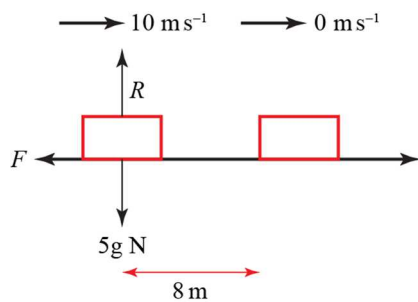
The K.E. gained is 16.2 J

b The work done by the force is 16.2 J

$$\begin{aligned} \text{c Work done} &= Fs \\ 16.2 &= F \times 4 \\ F &= \frac{16.2}{4} \\ F &= 4.05 \end{aligned}$$

The force has magnitude 4.05 N

12



$$\begin{aligned} \text{a K.E. lost} &= \frac{1}{2}mu^2 - \frac{1}{2}mv^2 \\ &= \frac{1}{2} \times 5 \times 10^2 - 0 \\ &= 250 \end{aligned}$$

The K.E. lost is 250 J

b Work done against friction = 250 J

$$\begin{aligned} \text{Work done} &= Fs \\ 250 &= F \times 8 \\ F &= \frac{250}{8} \end{aligned}$$

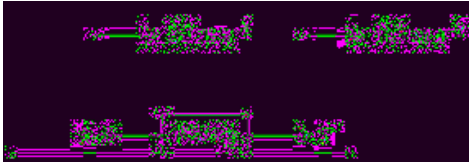
Resolving perpendicular to the slope:  $R = 5g$

Friction is limiting:  $F = \mu R$

$$\begin{aligned} \frac{250}{8} &= \mu \times 5g \\ \mu &= \frac{250}{8 \times 5g} \end{aligned}$$

The coefficient of friction is 0.638 (3 s.f.)

13



**a** Power =  $Fv$

$$15000 = T \times 20$$

$$T = \frac{15000}{20} = 750$$

Using  $F = ma$ :

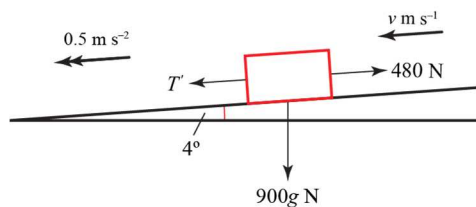
$$T - R = 900 \times 0.3$$

$$750 - R = 270$$

$$R = 750 - 270$$

$$R = 480$$

The magnitude of the resistance is 480 N

**b**

Resolving along the slope and using  $F = ma$ :

$$T' + 900g \sin 4^\circ - 480 = 900 \times 0.5$$

$$T' = 450 + 480 - 900g \sin 4^\circ$$

Power =  $Fv$

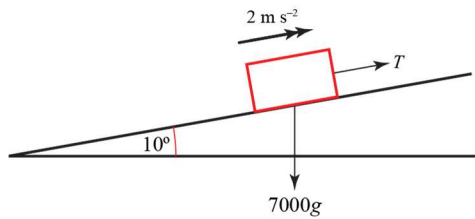
$$8000 = (450 + 480 - 900g \sin 4^\circ)v$$

$$v = \frac{8000}{(450 + 480 - 900g \sin 4^\circ)}$$

$$v = 25.41\dots$$

The speed of the car is  $25.4 \text{ m s}^{-1}$  (3 s.f.)

14



$$\text{Power} = Fv$$

$$\text{Power} = 4000 \text{ W}$$

$$T = \frac{4000}{v}$$

Resolving along the slope and using  $F = ma$ :

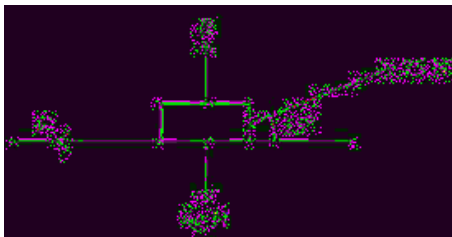
$$\frac{4000}{v} - 7000g \sin 10^\circ = 7000 \times 2$$

$$\frac{4000}{v} = 25912$$

$$\text{So } v = \frac{4000}{25912} = 0.154\dots$$

The speed of the bus is  $0.15 \text{ m s}^{-1}$  (2 s.f.)

15



$$\mu = \frac{3}{8}$$

**a** Resolving perpendicular to the floor:

$$R + 75 \sin 15^\circ = 4g$$

$$R = 4g - 75 \sin 15^\circ$$

Friction is limiting:

$$F = \mu R$$

$$F = \frac{3}{8} \times (4 \times 9.8 - 75 \sin 15^\circ)$$

$$F = 7.420\dots$$

The magnitude of the frictional force is  $7.42 \text{ N}$  (3 s.f.)

**b** Work done =  $F_s$

$$= 75 \cos 15^\circ \times 6$$

$$= 434.66\dots$$

The work done is  $435 \text{ J}$  (3 s.f.)

**15 c** Using the work–energy principle:

K.E. gained = work done by tension – work done against friction

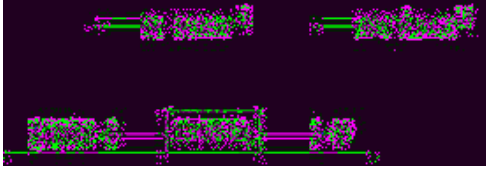
$$\frac{1}{2} \times 4v^2 = 434.66 - 7.420 \times 6$$

$$v^2 = \frac{1}{2}(434.66 - 7.420 \times 6)$$

$$v = 13.96 \dots$$

The block is moving at  $14.0 \text{ m s}^{-1}$  (3 s.f.)

**16 a**



At maximum speed,  $a = 0$

Resolving along the road and using  $F = ma$ :

$$T - 600 = 0$$

$$T = 600$$

Power =  $Fv$

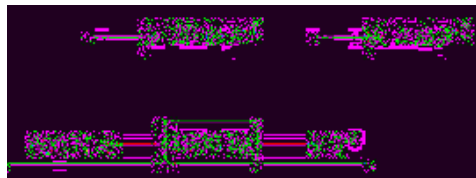
$$20000 = 600v$$

$$v = \frac{20000}{600}$$

$$v = 33.33$$

The lorry's maximum speed is  $33.3 \text{ m s}^{-1}$  (3 s.f.)

**b**



Power =  $Fv$

$$20000 = T' \times 20$$

$$T' = 1000$$

Using  $F = ma$ :

$$T' - 600 = 1800a$$

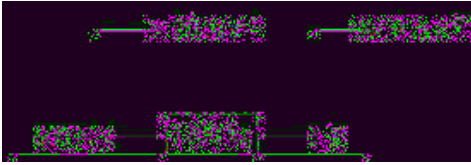
$$1000 - 600 = 1800a$$

$$a = \frac{400}{1800}$$

$$a = 0.2222 \dots$$

The acceleration of the lorry is  $0.222 \text{ m s}^{-2}$  (3 s.f.)

17

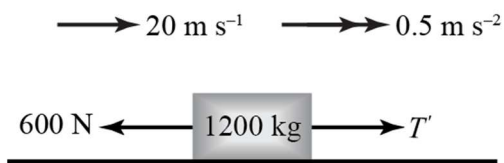


a Resolving along the road:  $T = 600$

$$\begin{aligned} \text{Power} &= Fv \\ &= 600 \times 20 \\ &= 12\,000 \text{ W} \\ &= 12 \text{ kW} \end{aligned}$$

The power is 12 kW

b



$$F = ma$$

$$T' - 600 = 1200 \times 0.5$$

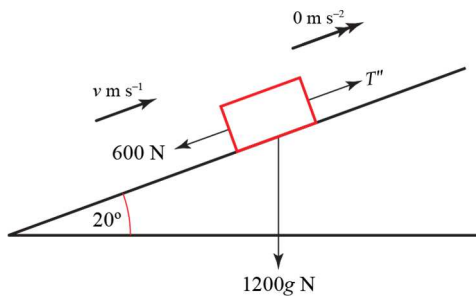
$$T' = 600 + 600$$

$$T' = 1200$$

$$\begin{aligned} \text{Power} &= F \times v \\ &= 1200 \times 20 \\ &= 24\,000 \end{aligned}$$

The new rate of working is 24 kW

c



Resolving along the slope:

$$T'' = 600 + 1200g \sin 20^\circ$$

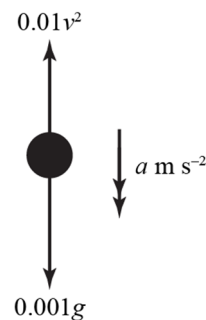
$$\text{Power} = Fv$$

$$50\,000 = (600 + 1200g \sin 20^\circ)v$$

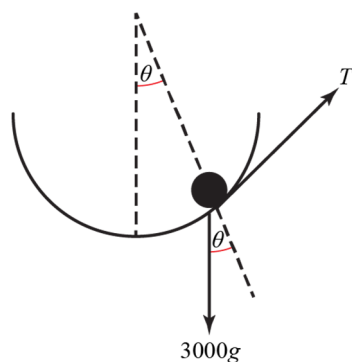
$$v = \frac{50\,000}{(600 + 1200g \sin 20^\circ)}$$

$$v = 10.82\dots$$

The value of  $v$  is 10.8 (3.s.f.)



## Challenge



- a** Car is moving with constant speed in a direction along the tangent to the cylinder.  
Resolving along the path of the car:

$$T = 3000g \sin \theta$$

$$\text{Power} = Tv$$

$$\text{Power} = 3000g \sin \theta \times 20 = 60\,000g \sin \theta = 588\,000 \sin \theta \text{ W}$$

- b** When  $\theta = 0^\circ$ , there is no force to act against, so no power is required.  
When  $\theta = 90^\circ$ , maximum power is needed.